Noncooperative Social Welfare Optimization With Resiliency Against Network Anomaly

Shengyi Wang , *Student Member, IEEE*, Liang Du , *Senior Member, IEEE*,
Jin Ye , *Senior Member, IEEE*, and Lina He, *Member, IEEE*

Abstract—This article presents a noncooperative game-theoretic framework to model the social welfare optimization (SWO) problem with load aggregators participating in integrated economic dispatch and demand response. In the proposed framework, distribution system operators interact with generation units and load aggregators to maximize the overall social welfare. The proposed SWO problem addresses practical system constraints and falls into the scope of mixed integer nonlinear programs, which cannot be well handled by existing distributed algorithms. The proposed SWO problem is formulated by a special noncooperative strategic game, known as the potential game, and solved by a revised version of the spatial adaptive play under network anomaly. It is shown that the proposed framework has guaranteed convergence to a Nash equilibrium that is also a global optimizer. Simulations on a 15-generator benchmark distribution network have been conducted to validate the proposed framework.

Index Terms—Constrained optimization, distributed algorithms, potential games, social welfare optimization (SWO).

Nomenclature

$\mathcal{G}$	Set of generators.
$\mathcal{E}$	Set of load aggregators.
C_i	Generation cost function of generator i.
U_i	Utility function of load aggregator i.
P_i	Power output when $i \in \mathcal{G}$/consumption when $i \in \mathcal{E}$.
η	Parameter for scaling the utility function.
P_i^{in}	Baseline need of load aggregator i.
$\widehat{P}_i$	Power level for load aggregator i.
n_i	Number of loads that load aggregator i manages.
P_D	Loads that do not participate in DR.

P^{loss}	Total transmission loss.
$\underline{P}_i$	Min. output when $i \in \mathcal{G}$/consumption when $i \in \mathcal{E}$.
$\overline{P}_i$	Max. output when $i \in \mathcal{G}$/consumption when $i \in \mathcal{E}$.
P_i^0	Previous power output of generator i.
DR_i	Down-ramp limit of generator i.
UR_i	Up-ramp limit of generator i.
k	Number of generators.
m	Number of load aggregators.
$\mathcal{F}_{i \in \mathcal{G}}$	Set of candidate power output for generator i.
$\mathcal{F}_{i \in \mathcal{E}}$	Set of load numbers that aggregator i can serve.
λ	Penalty multiplier to relax the equality constraint.
ϕ	Potential function.
u_i	Payoff function for player i.
T	Exploration parameter.
$\mathcal{J}(t)$	Set of channels with anomaly at t.
$P(t)$	Action profile at t.
$P_i(t)$	Action for player i at t.
$P_{-i}(t)$	Actions for players except player i at t.
$\mathcal{F}_i(t)$	Set of actions for player i at t.
$\hat{P}_i$	Trial action for player i.
z	Number of channels without anomaly.

I. Introduction

MODERNIZATION of electric power grids inherently consists of responsibilities and actions from both ends of supply and demand. On the supply end, system operators solve economic dispatch (ED) problems to procure a generation schedule of a specific time period by optimizing global objectives (often minimizing the total generation cost) with operation constraints [1]. On the demand end, demand response (DR) programs incentivize both commercial [2] and residential [3] end-users to control (often to reduce) their energy usage to maximize their own benefits [4] and improve grid reliability [5]. Both ED and DR are extensively studied constrained optimization problems with many solution algorithms available.

Conventionally, ED and DR are often considered separately. However, many recent works [6]–[11] have combined DR with ED into a unified framework, known as social welfare optimization (SWO). First discussed in [6], SWO addresses how DR-participating households maximize the social welfare under utilities' coordination. Furthermore, Samadi *et al.* [7] assumed two-way communications between end-users and utilities to share their demand information and maximize the social welfare. A consensus-based, cooperative algorithm is proposed

in [8] for distributed generations (DGs) and loads to maximize social welfare. Ma *et al.* [9] proposes an SWO model considering DC flow models solved by a two-layer mechanism including first consensus-based information discovery and then gradient-based generation. SWO is modeled as a convex problem with linear constraints and solved by dual decomposition in [10], while [11] considers linearized transmission losses, which constitute a nonconvex problem solved by the proposed transformation into convex subproblems.

Above-discussed SWO formulations are mostly based on simplified assumptions such as 1) transmission loss is decoupled or ignored; 2) DR units are comparable to generators in capacity; 3) cost functions are strictly increasing, convex, and smooth; and 4) communication is reliable and robust. However, in practice, cost functions are not always convex (e.g., if the valve-point effect is considered) or smooth (e.g., if multiple fuels are used or different incremental costs are present), and transmission loss is typically coupled and nonlinear. Existing SWO formulations and solutions cannot address these practical constraints and scenarios in terms of nonconvex, nonsmooth, or any cost functions. Therefore, this article introduces a novel SWO formulation with practical constraints, which can handle any formulations of cost and constraint.

Furthermore, power grids are experiencing a paradigm shift to incorporate high penetration of distributed energy resources, prosumers, and ubiquitous intelligent devices. Compared to conventional centralized decision-making frameworks, decentralized, autonomous, and self-interested decision-making better aligns with practical needs. Consensus-based and game-theoretic formulations are probably the two most popular such decision-making frameworks, in which each agent/player interacts with a defined group of other agents/players and makes decentralized, autonomous decisions. An overview of existing multiagent architectures for electric power grids can be found in [12]. One missing property in existing multiagent algorithms is that agents in general do not model self-interests of individuals, which can be well addressed by noncooperative game-theoretic formulations.

Moreover, above-discussed formulations involve many participants from geographically remote locations, and thus reliable and fast communication infrastructures are necessary. An overview of communication requirements for power grid applications is provided in [13], which points out that security and quality of service (QoS) concerns may arise when public networks are used for power grid applications. If the network is attacked, anomalous events that do not conform to expected normal behavior will be detected. In this article, *network anomaly* are considered to be unreliable communication conditions with low QoS and package drops. Possible causes of network anomaly can be harsh grid environment [14] or cyber-attacks on wireless networks, which are widely used in short to medium range power grid applications [15].

The impact of unreliable or limited communication on the performance of decentralized algorithms has been widely studied for decentralized control. Recent work shows that QoS [16], network topology changes [17], and communication delays [18] could cause cooperative control algorithms to fail. Moreover, a fallback control strategy is proposed in [19] for microgrids to mitigate denial-of-service network anomaly. However, to the authors' best knowledge, the performance of noncooperative games against network anomaly has not been well studied, with most of the literature focuses on communication delays [16], [20], [21]. Techniques such as the Artstein transformation [21] and model predictive control [20] are proposed to reduce the effect of time delays. However, the effect of frequent communication loss on decentralized control and optimization has not attracted much attention.

This article follows [22], [23] to integrate load aggregators into the conventional SWO and formulate the proposed SWO by a special noncooperative strategic game called the *potential game*. Each load aggregator [24] or generator is formulated as an independent and self-interested player who maximizes its own utility. The proposed formulation also considers many practical operating constraints such as ramp rates, prohibited zones, power balance, and load aggregator limits, which make the constrained SWO problem nonlinear and nonconvex and cannot be addressed by above-discussed SWO formulations. This article also assumes anomalous network conditions such that, at each time step, there is a random group of players that lose communication with others. A revised spatial adaptive play (SAP) named partial SAP (PSAP), with restricted action sets due to network anomaly, is proposed to solve this problem. Guaranteed convergence to the global optimum would be proved and validated in a widely used benchmark system.

The main contributions of this article are threefold.

1) Incorporating load aggregators and practical power flow constraints into the conventional SWO, which introduces a fundamentally different, nonlinear, and nonconvex SWO formulation.
2) Solving the proposed SWO in a noncooperative or even competitive manner through potential games. Each player is only self-interested, which reflects the fact that generating units and load units belong to many different owners and thus have quite different economic interests, which cannot be addressed by cooperative (though distributed) methods in literature.
3) Enhancing the resiliency of the proposed architecture against network anomaly with theoretically proven, guaranteed convergence to a Nash equilibrium (NE), which is also a global optimizer with arbitrarily high probability.

The remaining of this article is organized as follows. Section II defines utility functions, cost functions, and constraints of the proposed SWO problem. Section III introduces the concepts of potential games, the SAP, and a potential-game formulation of the proposed SWO problem. The PSAP and its convergence analysis in the proposed SWO problem are presented in Section IV. Section V presents numerical results on a 15-generator system and compares performances of the proposed PSAP algorithm in different communication environments. Finally, Section VI discusses extensions to nonsmooth formulations. Section VII concludes this article.

II. PROBLEM FORMULATION

In this section, the SWO problem considered in this article is presented. The main differences from other SWO formulations

in literature [7]–[11], [25] are twofold: 1) this article utilizes a different formulation as well as utility function for load aggregators, which induces additional mixed-integer property; and 2) this article considers widely adopted [26] practical constraints, which induces nonconvexity.

A. SWO Objective Function

The objectives of ED and DR considered in this article are to minimize the total generation cost $\sum_{i \in \mathcal{G}} C_i(P_i)$ and to maximize the overall benefits of DR-participating load aggregators $\sum_{i \in \mathcal{E}} U_i(P_i)$, respectively. Therefore, the overall objective is to maximize the following social welfare:

$$\max_{P_i} \sum_{i \in \mathcal{E}} U_i(P_i) - \sum_{i \in \mathcal{G}} C_i(P_i). \tag{1}$$

Let $\mathcal{G} := \{1, \ldots, k\}$ denote the set of generators. The generation cost function $C_i(P_i)$ in (1) is widely formulated [7]–[9], [11], [25] as follows:

$$C_i(P_i) = \alpha_i P_i^2 + \beta_i P_i + \gamma_i \tag{2}$$

where $P_i (i \in \mathcal{G})$ is the real power output of generator i, α_i, β_i, and γ_i are coefficients of the quadratic function.

Moreover, let $\mathcal{E} := \{k+1, \ldots, k+m\}$ denote the set of load aggregators, each of which manages DR-participating loads with the same power level and benefits from different types and models in an area. Load aggregators, such as direct controlled thermostat aggregators [27], electric vehicle (EV) aggregators [24], and residential load aggregators [28], have been proposed in many scenarios to participate in DR, day-ahead market, and real-time market.

Most existing work uses the linearly decreasing marginal DR utility function in SWO [7]–[11]. A sigmoid-type DR utility was presented in [29] for EV aggregators, with an indication parameter to represent range anxiety and thus EV drivers' desire to participate in DR. This article follows this concept and proposes the following sigmod-type utility function for load aggregator $i \in \mathcal{E}$ to incorporate their baseline needs to participate in DR:

$$U_i(P_i) = \frac{\eta}{1 + e^{-P_i + P_i^{\text{in}}}} \tag{3}$$

where P_i is the total consumption of the ith load aggregator, η is a predefined scaling parameter, and P_i^{in} is the baseline need of load aggregator i. It can be observed that in (3) load aggregator i receives only a small amount of utility when it reduces too much demand (actual demand P_i is small) and thus reflect end-users' satisfaction levels. Furthermore, the baseline demand can also be the real-time load forecasting and thus extended to incentives and real-time market operations. As shown in Fig. 1, when $P_i^{\text{in}} = 0$, it means that customers do not have interests to participate in DR, while $P_i^{\text{in}} > 0$ indicates that customers have incentives to participate DR with baseline P_i^{in}. For instance, it can be observed in Fig. 1 that "DR2" has more potential in demand reduction than "DR1".
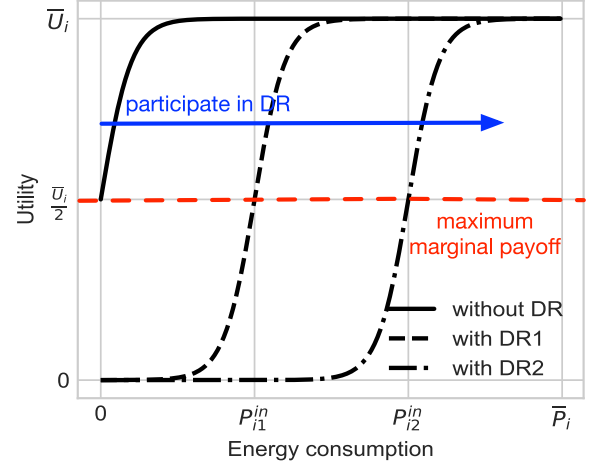

Fig. 1. Utility function of load aggregators, in which zero power consumption induces no DR utility.

B. SWO Operation Constraints

1) Load Aggregator Property: The demand of the ith load aggregator can be formulated as

$$P_i = n_i \widetilde{P}_i \qquad (n_i \in \mathcal{N}, i \in \mathcal{E}) \tag{4}$$

where n_i is the number of loads that the ith load aggregator manages, and $\widetilde{P}_i$ is its average baseline power level. Note that n_i is a variable in $P_i (i \in \mathcal{E})$ and thus needs to be optimized.

2) Active Power Balance:

$$\sum_{i \in \mathcal{G}} P_i = P_D + P^{\text{loss}} + \sum_{i \in \mathcal{E}} P_i \tag{5}$$

where P_D is the total demand except loads participating in DR (and thus not dispatchable). P^{loss} is the total transmission loss, which is generally estimated as a function of $P_i (i \in \mathcal{G})$ with Kron's B coefficients [30]–[32]

$$P^{\text{loss}} = \sum_{i \in \mathcal{G}} \sum_{j \in \mathcal{G}} P_i B_{ij} P_j + \sum_{i \in \mathcal{G}} B_{0i} P_i + B_{00}. \tag{6}$$

3) Generator Output Limits:

$$\underline{P}_i \leq P_i \leq \overline{P}_i \quad (i \in \mathcal{G}) \tag{7}$$

where $\underline{P}_i$ and $\overline{P}_i$ denote the lower and upper bounds of P_i.

4) Ramp Rate Limits:

$$P_i^0 - DR_i \leq P_i \leq P_i^0 + UR_i \quad (i \in \mathcal{G}) \tag{8}$$

where P_i^0, DR_i, and UR_i denote previous power output, down-ramp limit, and up-ramp limit of generator i, respectively.

5) Prohibited Zone Limits: Some thermal generators may not operate in the valve points and thus they should avoid zones, which contain those points. Feasible operation regions for generator i can be written as [30]–[32]

$$\begin{cases} \underline{P}_i \leq P_i \leq P_{i,1}^L \\ P_{i,s}^U \leq P_i \leq P_{i,s+1}^L, \quad i \in \mathcal{G}, s = 1, \ldots, N_{i-1} \\ P_{i,N_i}^U \leq P_i \leq \overline{P}_i \end{cases} \tag{9}$$
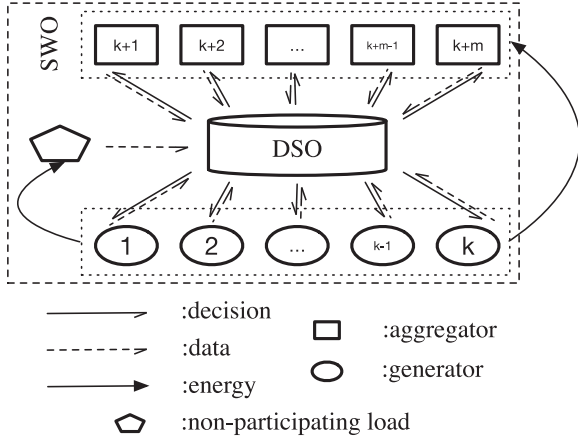
Fig. 2.   Implementation framework of proposed SWO.

where N_i is the total number of prohibited zones for i.

6) Load Aggregator Limits:

$$\underline{P}_i \leq P_i \leq \overline{P}_i \quad (i \in \mathcal{E}) \tag{10}$$

where $\underline{P}_i$ and $\overline{P}_i$ denote the lower and upper bounds of load aggregator i with actual demand P_i, respectively.

C. SWO Formulation in Mixed-Integer Nonlinear Programming (MINLP)

For $i \in \mathcal{G}$, define the set of candidate power outputs as

$$\mathcal{F}_{i \in \mathcal{G}} := \{P_i \in \mathbb{R} \mid \text{Constraints } 3), 4), 5)\}. \tag{11}$$

For $i \in \mathcal{E}$, define the set of candidate number of loads the aggregator i provides service to as

$$\mathcal{F}_{i \in \mathcal{E}} := \{n_i \in \mathbb{N} \mid \text{Constraints } 1) \text{ and } 6)\}. \tag{12}$$

Substituting $P_i (i \in \mathcal{E})$ in (1), (5), and (10) with n_i in (4) since n_i is to be solved, the SWO formulation can be written as

$$\max_{n_i, P_i} \sum_{i \in \mathcal{E}} U_i(n_i) - \sum_{i \in \mathcal{G}} C_i(P_i)$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{G}} P_i = P_D + P^{\text{loss}} + \sum_{i \in \mathcal{E}} n_i \widetilde{P}_i$$

$$P_i \in \mathcal{F}_{i \in \mathcal{G}}$$

$$n_i \in \mathcal{F}_{i \in \mathcal{E}}. \tag{13}$$

To summarize, the proposed SWO is an MINLP problem with both continuous and discrete variables, nonlinear objective functions, and nonlinear constraints. Fig. 2 shows the proposed SWO framework, in which a distribution system operators (DSO) collects load forecasting and generation data and broadcast to generation units and load aggregators. The SWO problem is solved in a decentralized manner by aggregators and generators whose are rational and self-interested. After the solution is achieved, each generator outputs as desired and each load aggregator manages its member loads to meet the assigned demand, respectively.

III. POTENTIAL GAME AND ITS FORMULATION OF SWO

In this section, the potential game [33] and the SAP learning algorithm are reviewed, followed by the potential-game formulation of the proposed SWO problem.

A. Noncooperative Strategic Games

A typical noncooperative strategic game consists of [22]
1) A set of players: $\mathcal{P} := \{1, \ldots, N\}$.
2) A set of actions for each player $i \in \mathcal{P}$: $\mathcal{A}_i$.
3) An action profile $a \in \mathcal{A} := \times_{i \in \mathcal{P}} \mathcal{A}_i$ is typically written as $a = (a_i, a_{-i})$, i.e., player i's action and everyone else's.
4) A payoff function for each player $i \in \mathcal{P}$: $u_i : \mathcal{A} \to \mathcal{R}$.
5) An action profile $a^* \in \mathcal{A}$ is a NE if and only if $u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*)$ for any $i \in \mathcal{P}, a_i \in \mathcal{A}_i$.
6) An action profile $a^* \in \mathcal{A}$ is a pure NE if $u_i(a_i^*, a_{-i}^*) = \max_{a_i \in \mathcal{A}_i} u_i(a_i, a_{-i}^*)$ for any $i \in \mathcal{P}$.

B. Potential Game

A potential game is a special noncooperative strategic game, in which the change in any player's utility function resulting from its unilateral change equals the change in a global utility named potential function. That is, for every player $\mathcal{P}_i$, for every $a_{-i} \in \mathcal{A}_{-i}$, and for every $a_i, a_i' \in \mathcal{A}_i$

$$u_i(a_i, a_{-i}) - u_i(a_i', a_{-i}) = \phi(a_i, a_{-i}) - \phi(a_i', a_{-i}). \tag{14}$$

If such a potential function $\phi : \mathcal{A} \to \mathbb{R}$ exists, this game is called a potential game with the potential function ϕ. Note that any action profile that maximizes the potential function is a pure NE of the potential game, and thus every potential game has at least one such NE. However, not every NE of a potential game maximizes its potential function. Thus, there could exist Nash equilibra that are only suboptimal [35, Section II.A], which are not desired solutions for engineering applications. Next, a learning algorithm that guarantees convergence to a NE that also maximizes the potential function is introduced.

C. Spatial Adaptive Play (SAP)

SAP is a learning algorithm in games, which can guarantee the convergence to a NE which is also the global optimizer with an arbitrarily high probability in potential games [34]. At each time step $t > 0$, one player $\mathcal{P}_i$ is randomly chosen (with equal probability for each player) and allowed to update its action. All other players must repeat their actions, i.e., $a_{-i}(t) = a_{-i}(t-1)$. The updating player $\mathcal{P}_i$ randomly selects an action $a_i \in \mathcal{A}_i$ according to the softmax distribution $Pr_i(t) \in \Delta(\mathcal{A}_i)$ of which the a_ith component $Pr_i^{a_i}(t)$ is given as

$$Pr_i^{a_i}(t) = \frac{\exp\{T u_i(a_i, a_{-i}(t-1))\}}{\sum_{a_i' \in \mathcal{A}_i} \exp\{T u_i(a_i', a_{-i}(t-1))\}} \tag{15}$$

where $T \geq 0$ is the exploration parameter, and $\Delta(\mathcal{A}_i)$ denotes the set of all possible probability distributions over the set $\mathcal{A}_i$. Note that T determines how likely player $\mathcal{P}_i$ selects a suboptimal action. If $T = 0$, player $\mathcal{P}_i$ selects any action with equal probability. If $T \to \infty$, player $\mathcal{P}_i$ selects an action from its

best response set with arbitrarily high probability. Therefore, if all players update their actions following SAP with sufficiently large t and T, then the players will reach an NE. Furthermore, such an NE is a global optimizer, which maximizes the potential function [35].

D. Potential-Game Formulation of SWO

The proposed SWO problem can be formulated as a potential game with load aggregators and generators considered as self-interested players, whose actions are demands and generations, respectively. The objective function can be rewritten as follows with a penalty multiplier λ adopted to relax the equality constraint:

$$\max_{n_i, P_i} \sum_{i \in \mathcal{E}} U_i(n_i) - \sum_{i \in \mathcal{G}} C_i(P_i)$$
$$- \lambda \left| P_D + P^{\text{loss}} + \sum_{i \in \mathcal{E}} n_i \widetilde{P}_i - \sum_{i \in \mathcal{G}} P_i \right| \quad (16)$$

where the penalty multiplier λ should be positively large so that the power mismatch is then driven to approach zero.

In this article, the potential function is designed as the objective function in (16). For convenience of notation and without loss of generality, n_i is replaced by $P_i (i \in \mathcal{E})$ here for notation conveniences in the following theoretical derivation. Thus, the potential function is written as:

$$\phi(P_i, P_{-i}) = \sum_{i \in \mathcal{E}} U_i(P_i) - \sum_{i \in \mathcal{G}} C_i(P_i)$$
$$- \lambda \left| P_D + P^{\text{loss}} + \sum_{i \in \mathcal{E}} P_i - \sum_{i \in \mathcal{G}} P_i \right|. \quad (17)$$

The payoff function for each player is designed to be

$$u_i(P_i, P_{-i}) = -(\alpha_i P_i^2 + \beta_i P_i)$$
$$- \lambda \left| P_D + P^{\text{loss}} + \sum_{i \in \mathcal{E}} P_i - \sum_{i \in \mathcal{G}} P_i \right| \quad (i \in \mathcal{G})$$
$$(18)$$

$$u_i(P_i, P_{-i}) = U_i(P_i)$$
$$- \lambda \left| P_D + P^{\text{loss}} + \sum_{i \in \mathcal{E}} P_i - \sum_{i \in \mathcal{G}} P_i \right| \quad (i \in \mathcal{E}).$$
$$(19)$$

A potential game is formed by the SWO problem with potential function ϕ. Details of the proof can be found in [23] and omitted in this article due to space limit. If all players (aggregators and generators) stick with SAP with perfect communication environment, the interacting process is guaranteed to converge to an NE, which is also a global maximizer to the potential function as well as the relaxed SWO objective function [23].

Algorithm 1: PSAP by DSO.

1: Initialize $P_i(t)$
2: **repeat**
3:     At time t, detect the set of low-QoS channels $\mathcal{J}(t)$ that failed to communicate at t
4:     Randomly select $i \in \mathcal{G} \cup \mathcal{E}$
5:     **if** $i \in \mathcal{J}(t)$ **then**
6:         Assign $P_i(t) \leftarrow P_i(t-1)$
7:     **else**
8:         Notify i and send $P_{-i}(t-1)$ to i
        (Go to **Algorithm 2**: 1)
9:         Receive $P_i(t)$ from i
10:        Assign $P_{-i}(t) \leftarrow P_{-i}(t-1)$
11:     **end if**
12: **until** Converge to NE or STOP signal

IV. SAP UNDER NETWORK ANOMALY

The communication architecture considered in this article is shown in Fig. 2. There are other communication architectures available in literature for SWO such as peer-to-peer communications. However, it is more practical to have DSOs (which are heavily protected under regulations for Critical Infrastructures) to handle all information exchange instead of having geographically remote and self-interested players keep location and high-fidelity information about others.

As (15) shows, for every player to update with SAP at each time step t, it needs to have full information of $a_{-i}(t-1)$, i.e., all the actions of others from last time step. Therefore, the conventional SAP will not perform in a network with anomaly. This article proposes a revised version of SAP, called *PSAP*, since, at each time step, it is assumed that there is a *partial* group randomly selected players who cannot communicate to the DSO and thus cannot receive the most updated actions from others.

In the proposed PSAP, the DSO does not make any decisions, so its role is to evaluate channels, collect, and broadcast information. Algorithm 1 operates in the manner that at each time step the DSO first pings all channels and determines which channels have anomaly (e.g., low QoS, network topology changes, or communication delays). Since one channel can be jammed for a certain amount of time, at each time step, the DSO then only communicate the last-successfully communicated actions from those jammed channels. In other words, the set of action profiles for PSAP is only a subset of the conventional SAP with a "partial" group of players participating at each time step.

Similar to traditional SAP, the DSO also randomly selects a player i with equal probability to update its action. If the selected player is not reachable due to network anomaly, i.e., it is in $\mathcal{J}(t)$, then the DSO wait until next time step. Otherwise, the DSO notifies it to play SAP according to $P_{-i}(t-1)$ sent by the DSO. Note that $P_{-i}(t-1)$ consists of last known actions of all players except the updating player. For each player i, it only updates when it receives a notification from the DSO. Otherwise,

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Algorithm 2: PSAP by Each Player $\mathcal{P}_i$.

1: Notified by the DSO to update with $P_{-i}(t-1)$
2: **if** $i \in \mathcal{G}$ **then**
3:     Update $P_i(t) \sim softmax[u_i(P_i, P_{-i}(t-1)), T]$
       where $P_i \in \mathcal{F}_{i \in \mathcal{G}}$
4: **else**
5:     Update $P_i(t) \sim softmax[u_i(P_i, P_{-i}(t-1)), T]$
       where $P_i \in \mathcal{F}_{i \in \mathcal{E}}$
6: **end if**
7: Notify DSO and send $P_i(t)$ to DSO
    (Go back to **Algorithm 1**: 9)

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no matter what status the corresponding communication channel has, all players repeat their last action.

The convergence analysis of the proposed PSAP is shown as follows. Denote an action profile at t by $P(t)$ as a random variable, which consists of $k + m$ elements, a set of actions for every player i at t can then by denoted by $\mathcal{F}_i(t)$ where $P_i(t) \in \mathcal{F}_i(t)$ and $P(t) \in \times_{i \in \mathcal{G} \cup \mathcal{E}} \mathcal{F}_i(t)$. When player i is not experiencing network anomaly, i.e., $i \notin \mathcal{J}(t)$, the set of actions available to player i at time t is then

$$\mathcal{F}_i(t) = \begin{cases} \mathcal{F}_{i \in \mathcal{G}} & \text{if } i \in \mathcal{G} \\ \mathcal{F}_{i \in \mathcal{E}} & \text{if } i \in \mathcal{E} \end{cases}. \qquad (20)$$

Otherwise, i.e., $i \in \mathcal{J}(t)$, the set of actions available to player i at time t is

$$\mathcal{F}_i(t) = \{P_i(t-1)\}. \qquad (21)$$

Equations (20) and (21) are restricted action sets. Similar to [35], the updating player randomly selects a trial action $\hat{P}_i$ from its restricted action set with the following probabilities. Let $z = k + m - |\mathcal{J}(t)|$ denote the number of channels without network anomaly conditions, and the probability that the $\hat{P}_i$ is selected is given by

$$Pr[\hat{P}_i \in \mathcal{F}_i(t), \quad i \notin \mathcal{J}(t)] = \frac{z}{k+m}$$

$$Pr[\hat{P}_i \in \mathcal{F}_i(t), \quad i \in \mathcal{J}(t)] = \frac{|\mathcal{J}(t)|}{k+m}. \qquad (22)$$

After player i selects $\hat{P}_i$, the player chooses its action at time t according the following probability:

$$Pr[P_i(t) = \hat{P}_i] = \frac{\exp\{Tu_i(\hat{P}_i, P_{-i}(t-1))\}}{\sum_{P_i \in \mathcal{F}_i(t)} \exp\{Tu_i(P_i, P_{-i}(t-1))\}}$$

$$Pr[P_i(t) = P_i(t-1)] = 1 \qquad (23)$$

where the first equation in (23) corresponds to Steps 3 and 5 in **Algorithm 2**, and the second equation in (23) corresponds to Step 6 in **Algorithm 1**.

Furthermore, given a discrete-time Markov chain (DTMC) with transition matrix $P = [p_{ij}]$, an equilibrium distribution μ is said to be in *detailed balance* if $\mu_i p_{ij} = \mu_j p_{ji}$ for all $i, j \in \mathcal{S}$ [36]. Moreover, μ is a *stationary distribution* of the DTMC

since $\sum_i \mu_i p_{ij} = \sum_i \mu_j p_{ji} = \mu_j \sum_i p_{ji} = \mu_j$ and therefore $\mu = \mu P$.

The following theorem states that, within the potential-game formulated SWO, the proposed PSAP induces a DTMC over state space $P(t)$ with a unique stationary distribution.

Theorem 1: Consider a finite $(k + m)$-player potential game with the potential function $\phi(\cdot)$. If a DTMC $\{P(t), t \geq 0\}$ induced by the proposed PSAP over the state space $\mathcal{S} := \times_{i \in \mathcal{G} \cup \mathcal{E}} \mathcal{F}_i(t)$ is irreducible and aperiodic, and it has the unique stationary distribution given by

$$\mu(p) = \frac{\exp\{T\phi(p)\}}{\sum_{\bar{p} \in \mathcal{S}} \exp\{T\phi(\bar{p})\}} \quad \text{for any} \quad p \in \mathcal{S}. \qquad (24)$$

Proof: For any $p, p' \in \mathcal{S}$

$$p_{pp'} := Pr[P(t) = p'|P(t-1) = p]. \qquad (25)$$

Since player i has probability $\frac{1}{k+m}$ of being chosen in any given period and has probability $\frac{z}{k+m}$ of any trial action $\hat{P}_i$ selected without network anomaly, it follows that:

$$\mu(p)p_{pp'} = \left[\frac{\exp\{T\phi(p)\}}{\sum_{\bar{p} \in \mathcal{S}} \exp\{T\phi(\bar{p})\}} \right]$$

$$\times \left[\frac{1}{k+m} \frac{z}{k+m} \frac{\exp\{Tu_i(p', P_{-i}(t-1))\}}{\sum_{P_i \in \mathcal{F}_i(t)} \exp\{Tu_i(P_i, P_{-i}(t-1))\}} \right] \qquad (26)$$

$$\mu(p')p_{p'p} = \left[\frac{\exp\{T\phi(p')\}}{\sum_{\bar{p} \in \mathcal{S}} \exp\{T\phi(\bar{p})\}} \right]$$

$$\times \left[\frac{1}{k+m} \frac{z}{k+m} \frac{\exp\{Tu_i(p, P_{-i}(t-1))\}}{\sum_{P_i \in \mathcal{F}_i(t)} \exp\{Tu_i(P_i, P_{-i}(t-1))\}} \right]. \qquad (27)$$

Let

$$\pi = \frac{1}{\sum_{\bar{p} \in \mathcal{S}} \exp\{T\phi(\bar{p})\}} \frac{\frac{z}{(k+m)^2}}{\sum_{P_i \in \mathcal{F}_i(t)} \exp\{Tu_i(P_i, P_{-i}(t-1))\}} \qquad (28)$$

then

$$\mu(p)p_{pp'} = \pi \exp\{T\phi(p) + Tu_i(p', P_{-i}(t-1))\}. \qquad (29)$$

Since

$$u_i(p', P_{-i}(t-1)) - u_i(p, P_{-i}(t-1)) = \phi(p') - \phi(p) \qquad (30)$$

it leads to

$$\mu(p)p_{pp'} = \pi \exp\{T\phi(p') + Tu_i(p, P_{-i}(t-1)\} \qquad (31)$$

and

$$\mu(p)p_{pp'} = \mu(p')p_{p'p}. \qquad (32)$$

The detailed balance condition is then established. It follows immediately that μ is a stationary distribution of the DTMC $\{P(t), t \geq 0\}$. Given the state-space $\mathcal{S}$, the process in any period is irreducible, i.e., all states communicate with each other, and aperiodic. Therefore, it has a unique stationary distribution which must be μ. This completes the proof of Theorem 1. ∎

TABLE I
SOLUTIONS BY PSAP AND TRADITIONAL SAP

i	P_i^{in}	$P_{i \in \mathcal{G}}$(w/o)	n_i(w/o)	$P_{i \in \mathcal{G}}$(w/)	n_i(w/)
1	13	449.68	999	452.56	1115
2	12.5	378.97	1235	379.56	1250
3	13	128.06	738	122.10	866
4	9	128.12	57	129.83	48
5	10	167.99	64	169.23	99
6	10	459.33	99	456.35	76
7	6	429.65	35	429.16	53
8	8	157.42	70	154.65	59
9	10	149.64	97	158.01	64
10	9	155	59	155.00	88
11	10	77.32	91	78.20	97
12	9	75.75	89	73.95	71
13	10	80.16	58	81.50	82
14	8	47.73	48	49.15	51
15	10	49.44	88	51.08	97

Corollary 1: For every monotonically increasing exploration parameter $T(t) > 0$, i.e., $T(t') > T(t)$ if $t' > t$, then Theorem 1 still holds.

Proof: It can be observed that (32) holds with only denominators containing T terms in (26)–(28) cancel out on both sides of (32). This completes the proof of Corollary 1. ∎

From Theorem 1, the unique stationary distribution $\mu(p)$ is an instance of Gibbs distribution. For sufficiently large times $t > 0$, $\mu(p)$ is equal to the probability that $P(t) = p$. As $T \to \infty$, $P(t)$ follows the unique p with arbitrarily high probability $\mu(p)$ such that p as an NE maximizes the potential function. So the objective function in (16) is maximized as well. It is noted that the final action profile p can be different in the different communication environment.

V. NUMERICAL RESULTS

In this section, the effectiveness of the proposed PSAP in different communication environment is validated in a widely used benchmark distribution system [22], [26], [30]–[32] with 15 generators. Modifications are made to include participation of three load aggregators (10 kW) and 12 load aggregators (100 kW). Generators parameters can be found in above references and, thus, are omitted here due to space limit.

The solutions to the SWO (with near-full loading) shown in Table I and Fig. 3 are based on $\lambda = 3000$ and $\eta = 15$. One solution is the proposed PSAP with a random number of channels selected with network anomaly (shown in Red), and the other solution is by traditional SAP with full communication without network anomaly (shown in Blue).

It can be observed that
1) Fig. 3(a) shows that the global potential function increases quickly and converges to a near steady final value after around 50 iterations in both cases.
2) Fig. 3(b) shows that the total social welfare converges along with the global potential function in both cases.

Note that with network anomaly, the social welfare is lower, due to restricted action sets caused by limited choices of updating players.
3) Fig. 3(c) shows that the total utility of all aggregators decreases along with the decrease of the total energy consumption, which is shown in Fig. 3(e).
4) Fig. 3(d) shows that the total generation cost drops along with the decrease of the total energy consumption shown in Fig. 3(e) as well as the total generation shown in Fig. 3(i). Note that with network anomaly, the generation cost is higher, due to the same reason mentioned in Fig. 3(b).
5) Fig. 3(e) shows that load aggregators have participated in DR in terms of reducing total energy consumption, although it decreases loads' total utility. Note that with network anomaly, the total energy consumption is higher.
6) Fig. 3(f) shows the player which is randomly selected at each iteration of the PSAP learning process.
7) Fig. 3(g) shows that the total generation drops as load aggregators participating in DR to reduce peak demand. Note that with network anomaly the total generation is higher due to the same reason as Fig. 3(b).
8) Fig. 3(h) shows that the total transmission loss converges. Note that with network anomaly the total transmission loss is higher due to the same reason as in Fig. 3(b).
9) Fig. 3(i) shows that the total load/demand drops as load aggregators participate in DR.
10) Fig. 3(j) shows that the power balancing converges.

To summarize, under network anomaly, the proposed PSAP can converge as expected. The overall convergence is slower compared to the scenarios without communication issues due to limited choices of updating players at each step. Also note that due to limited information and choices at each time step, the total energy cost might be slightly higher under network anomaly but the overall global utilities converge to a value almost identical to the scenarios without network anomaly, as shown in Fig. 3(a) and (c).

It is also noted that the setting of η is critical to the performance of PSAP. An appropriate setting should satisfy the following condition [23]:

$$\eta < 4 \min \{2\alpha_i \underline{P}_i + \beta_i, \ i \in \mathcal{G}.\}. \tag{33}$$

VI. EXTENSIONS TO NONSMOOTH FORMULATIONS

Note that in the potential game framework, the cost, utility, or potential functions can be nonsmooth, nonconvex or of any form. In [37], potential games are applied to plug-in hybrid EVs charging problem with concave utility functions. In [38], potential games are applied to solve the mathematical puzzle Sudoku with nonsmooth utilities. Furthermore, the proposed SWO formulation can also be inherently extended to nonsmooth cost objective functions. Consider the cases in which multiple fuels are used and the objective function can be expressed as the
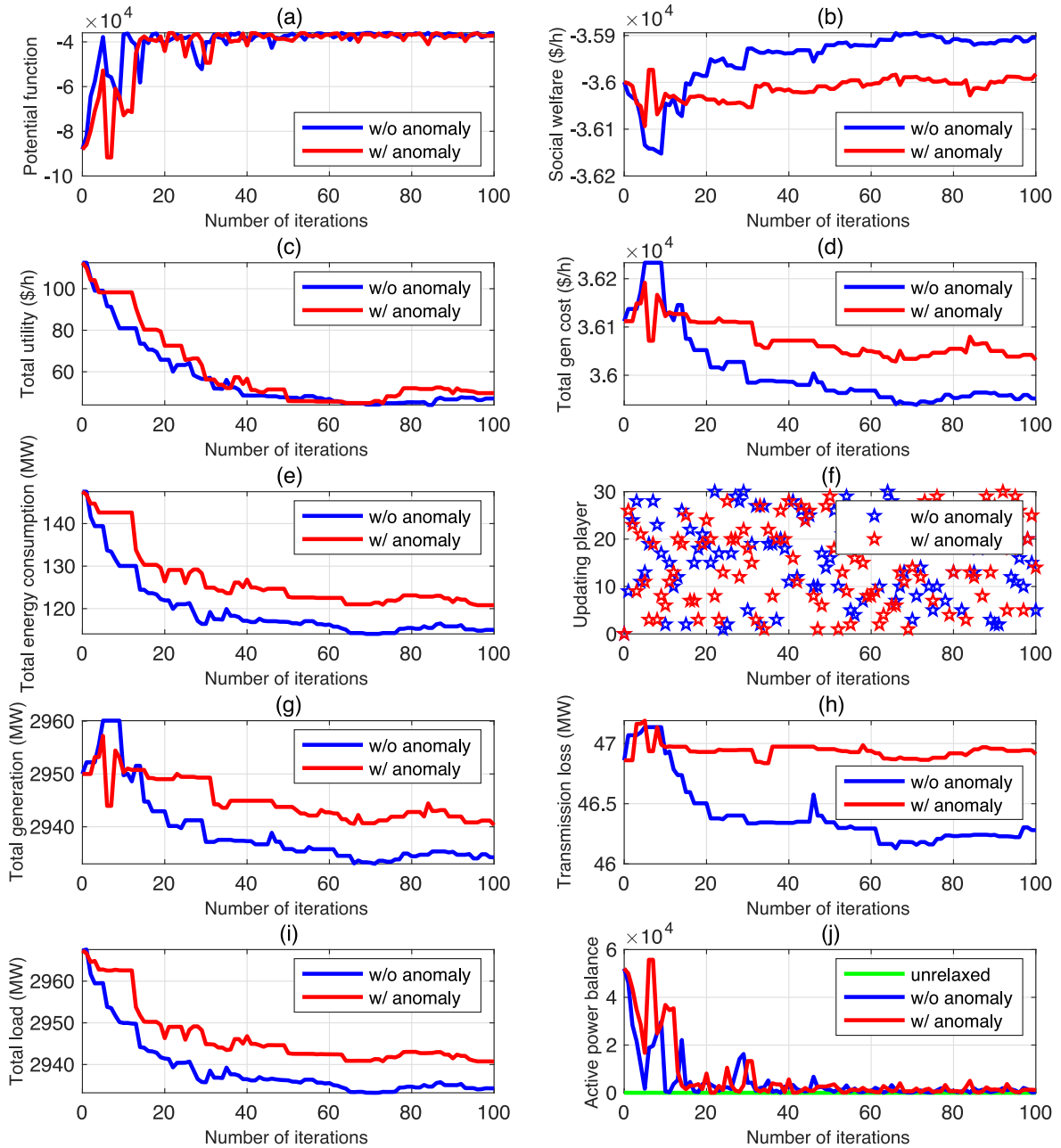
Fig. 3.   Update of all indicators under the near full-load scenario.

following piece-wise quadratic cost function:

$$
C_i\left(P_i\right) =
\begin{cases}
\alpha_{i,1} + \beta_{i,1}P_i + \gamma_{i,1}P_i^2, \\
\qquad \text{if } P_{i,\min} \leq P_i \leq P_{i,1} \\
\alpha_{i,2} + \beta_{i,2}P_i + \gamma_{i,2}P_i^2, \\
\qquad \text{if } P_{i,1} \leq P_i \leq P_{i,2} \\
\alpha_{i,n} + \beta_{i,n}P_i + \gamma_{i,n}P_i^2, \\
\qquad \text{if } P_{i,n-1} \leq P_i \leq P_{i,\max}
\end{cases}
$$

which is composed of a finite number of subproblems, each of which falls into the proposed formulation. Therefore, the

discontinuous cost functions can also be effectively handled by the proposed formulation.

VII. CONCLUSION

This article took into consideration the impact of network anomaly on a noncooperative formulation of the constrained SWO problem. Continuing with the previously proposed potential-game formulation of the SWO, a variant of the SAP algorithm called PSAP was proposed and analyzed. With all players stick to the proposed PSAP, it was shown that the induced DTMC guarantees to converge to an NE, which is also a global maximizer with arbitrarily high probability.

Numerical simulations of SWO with network anomaly solved by the proposed PSAP and SWO without network anomaly solved by conventional SAP were presented and compared, with results outcomes met expectations. For future work, the proposed algorithm can be improved with more effective way to handle the power balance equality constraints. Also, the proposed algorithm can be extended to vector-based action profiles to consider a long-duration SWO problem.

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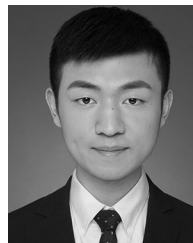
Shengyi Wang (S'17) received the B.S. and M.S. degrees in electrical engineering from the Shanghai University of Electric Power, Shanghai, China, and Clarkson University, Potsdam, NY, USA, in 2016 and 2017, respectively. He is currently working toward the Ph.D. degree in electrical and computer engineering with Temple University, Philadelphia, PA, USA.

His research interests include game-theoretic control for multiagent systems, data-driven optimization for power systems, and nonintrusive load monitoring.

Dr. Wang received Outstanding Undergraduate Thesis Award in 2016.

Liang Du (S'09–M'13–SM'18) received the Ph.D. degree in electrical engineering from the Georgia Institute of Technology, Atlanta, GA, USA, in 2013.

He was a Research Intern with Eaton Corp. Innovation Center, Milwaukee, WI, USA, Mitsubishi Electric Research Labs, Cambridge, MA, USA, and Philips Research N. A., Briarcliff Manor, NY, USA, in 2011, 2012, and 2013, respectively. He was also an Electrical Engineer with Schlumberger, Sugar Land, TX, USA, from 2013 to 2017. He is currently an Assistant Professor with Department of Electrical and Computer Engineering, Temple University, Philadelphia, PA, USA.

Dr. Du received the Ralph E. Powe Junior Faculty Enhancement Award from Oak Ridge Associated Universities (ORAU) in 2018 and currently serve as an Associate Editor for IEEE TRANSACTIONS ON INDUSTRY APPLICATIONS.

Lina He (S'11–M'15) received the B.S. and M.S. degrees in electrical engineering from the Huazhong University of Science and Technology, Wuhan, China, in 2007 and 2009, respectively, and the Ph.D. degree in electrical engineering from the University College Dublin, Dublin, Ireland, in 2014.

She is currently an Assistant Professor with the Department of Electrical and Computer Engineering, University of Illinois, Chicago, IL, USA. She was a Project Manager and Senior Consultant with Siemens from 2014 to 2017. Her research interests include renewable energy integration, power systems and coordination with power electronics, wide-area protection, and cybersecurity.

Jin Ye (S'13–M'14–SM'16) received the B.S. and M.S. degrees from Xi'an Jiaotong University, Xi'an, China, in 2008 and 2011, respectively. She received the Ph.D. degree from McMaster University, Hamilton, ON, Canada, in 2014, all in electrical engineering.

She is currently an Assistant Professor with the School of Electrical and Computer Engineering, University of Georgia, Atlanta, GA, USA.

Dr. Ye is a General Chair of 2019 IEEE Transportation Electrification Conference and Expo, a Publication Chair and Women in Engineering Chair of 2019 IEEE Energy Conversion Congress and Expo. She is an Associate Editor for IEEE TRANSACTIONS ON TRANSPORTATION ELECTRIFICATION and IEEE TRANSACTION ON VEHICULAR TECHNOLOGY.